MECHANICS OF MATERIALS

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Concept of Stress

- The main objective of the study of the strength of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.

- Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

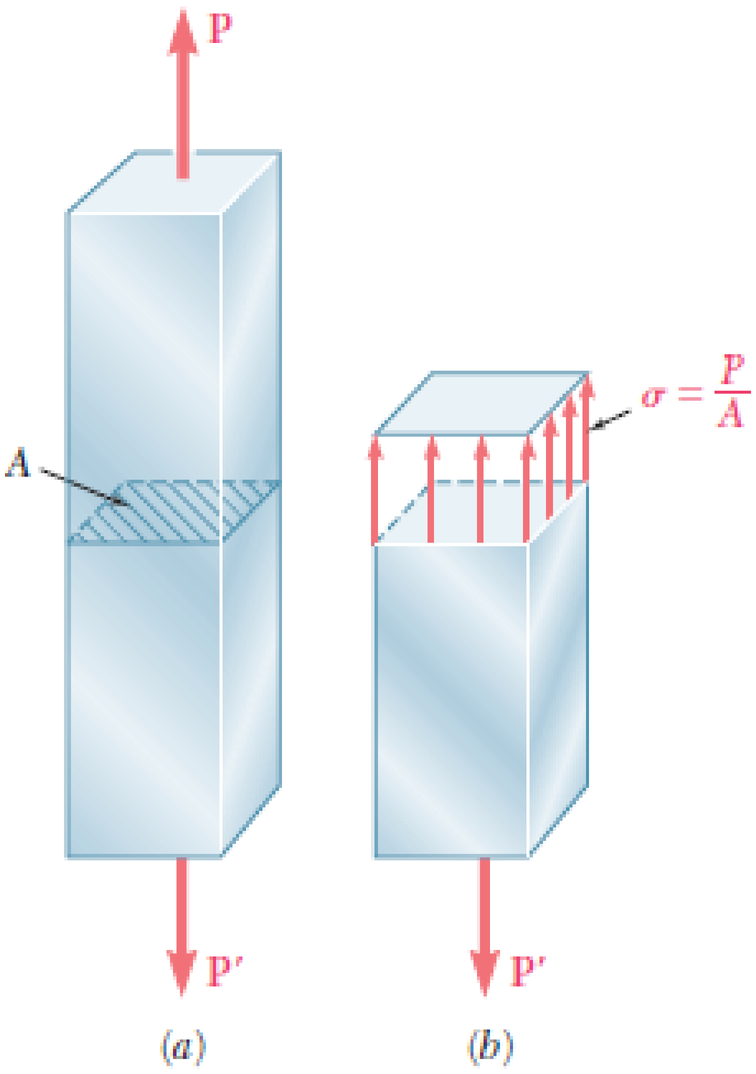


Fig. 1.1 Member with an axial load.

- The force per unit area, or intensity of the forces distributed over a given section, is called the stress on that section and is denoted by Greek letter **σ** (sigma) (Fig. 1.1).

- The stress in a member of cross-sectional area A subjected to an axial load **P** is therefore obtained by

$$\sigma = \frac{P}{A}$$

- Positive sign will be used to indicate a tensile stress (member in tension) and negative sign to indicate a compressive stress (member in compression)

- Stress is expressed in SI units as below,

$$1\ \text{kPa} = 10^3\ \text{Pa} = 10^3\ \text{N/m}^2$$

$$1\ \text{MPa} = 10^6\ \text{Pa} = 10^6\ \text{N/m}^2$$

$$1\ \text{GPa} = 10^9\ \text{Pa} = 10^9\ \text{N/m}^2$$

Stress Analysis

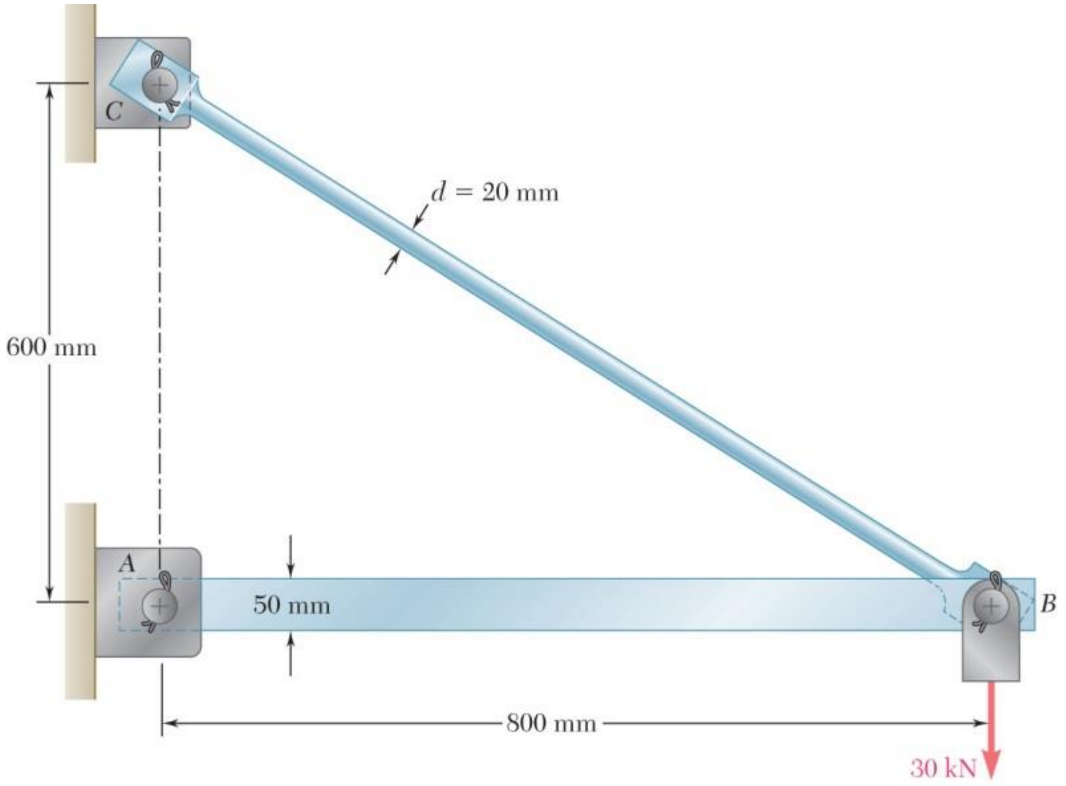
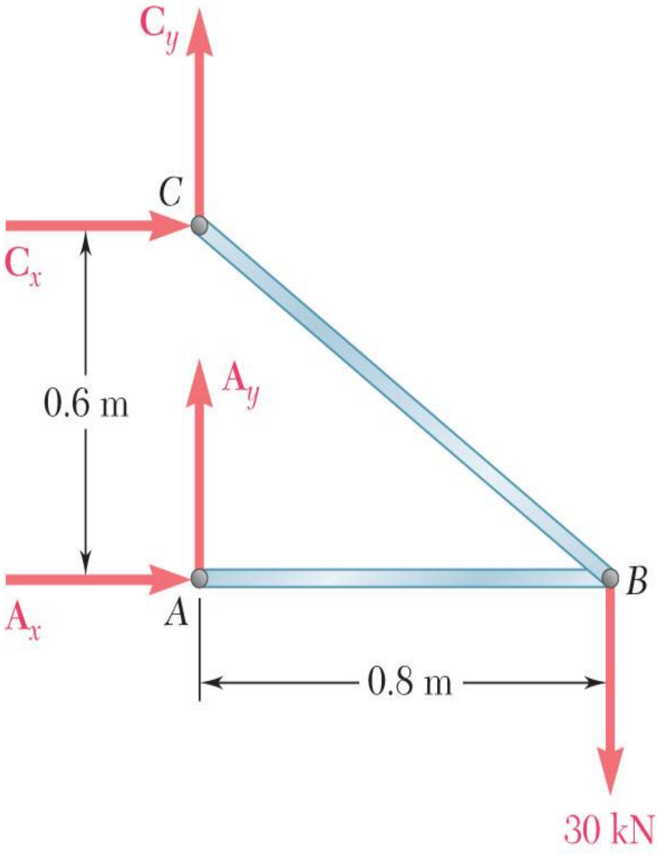


Fig. 1.2 Boom used to support a 30-kN load.

Rod *BC* is made of a steel with a maximum allowable stress σ_{all}=165 MPa. Can the structure safely support the 30 kN load if rod *BC* has a diameter of 20 mm?



- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at the ends of the members

- Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0{,}6\,\text{m}) - (30\,\text{kN})(0{,}8\,\text{m})$$

$$A_x = +40\,\text{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

$$C_x = -A_x = -40\,\text{kN}$$

$$\sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$

$$A_y + C_y = +30\,\text{kN}$$

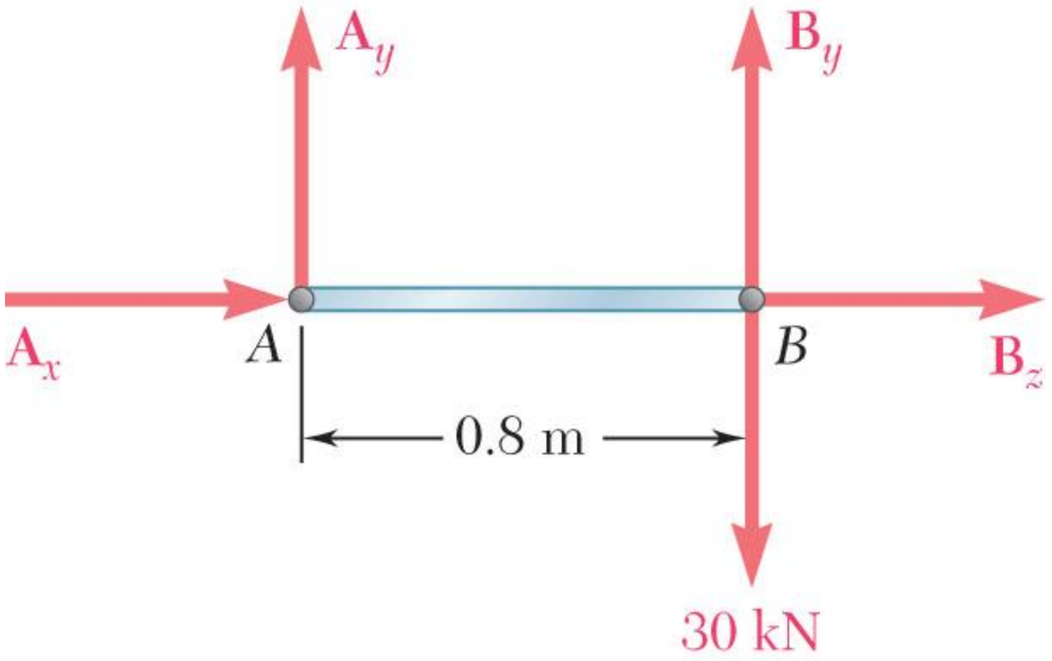


Fig. 1.3 Free-body diagram of member *AB* freed from structure.

- In addition to the complete structure, each component must satisfy the conditions for static equilibrium

- Consider a free-body diagram of the boom *AB*:

$$+\circlearrowleft \sum M_B = 0 = -A_y(0{,}8\,\text{m})$$

$$A_y = 0$$

substitute into the structure equilibrium equation

$$C_y = +30\,\text{kN}$$

- Results:

$$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along the boom and rod

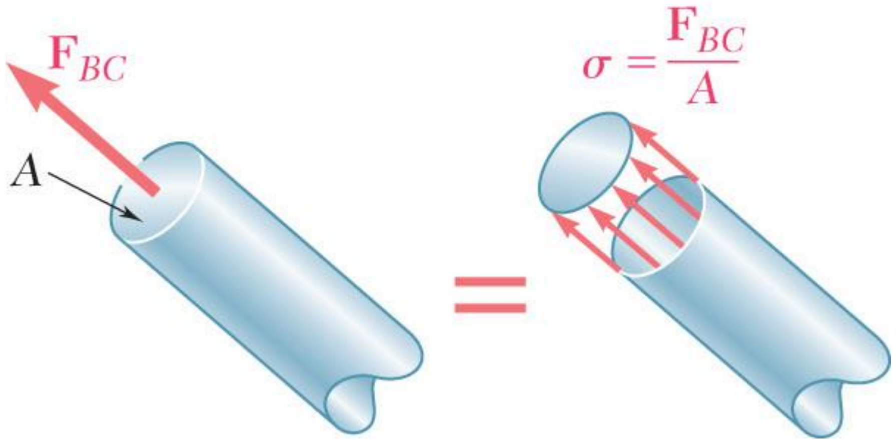


Fig. 1.7 Axial force represents the resultant of distributed elementary forces.

- At any section through member BC, the internal force is 50 kN with a force intensity or <u>stress</u> of

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3\,\mathrm{N}}{314 \times 10^{-6}\,\mathrm{m}^2} = 159\ \mathrm{MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\mathrm{all}} = 165\,\mathrm{MPa}$$

- Conclusion: the strength of member *BC* is adequate

Design

- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^{3}\,\mathrm{N}}{100\times10^{6}\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

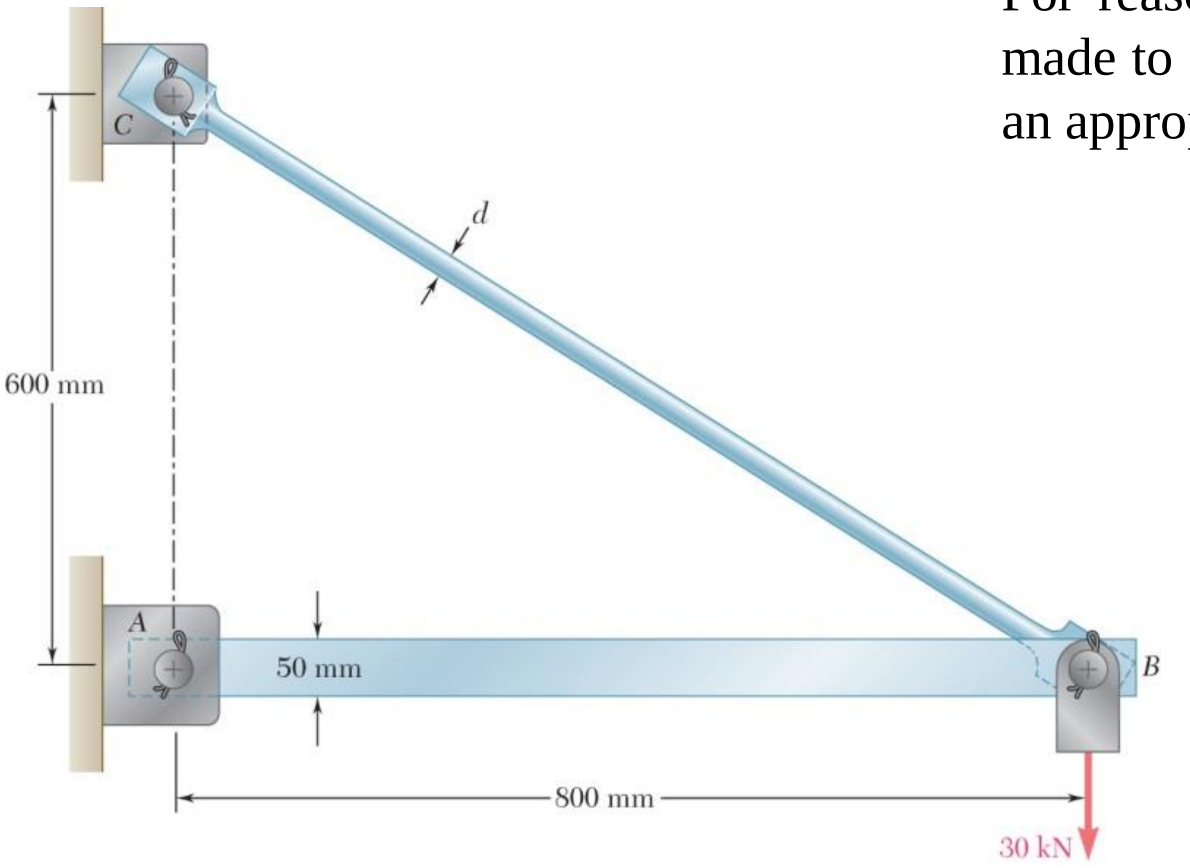


Fig. 1.1 Boom used to support a 30-kN load.

- An aluminum rod 26 mm or more in diameter is adequate

Axial Loading: Normal Stress

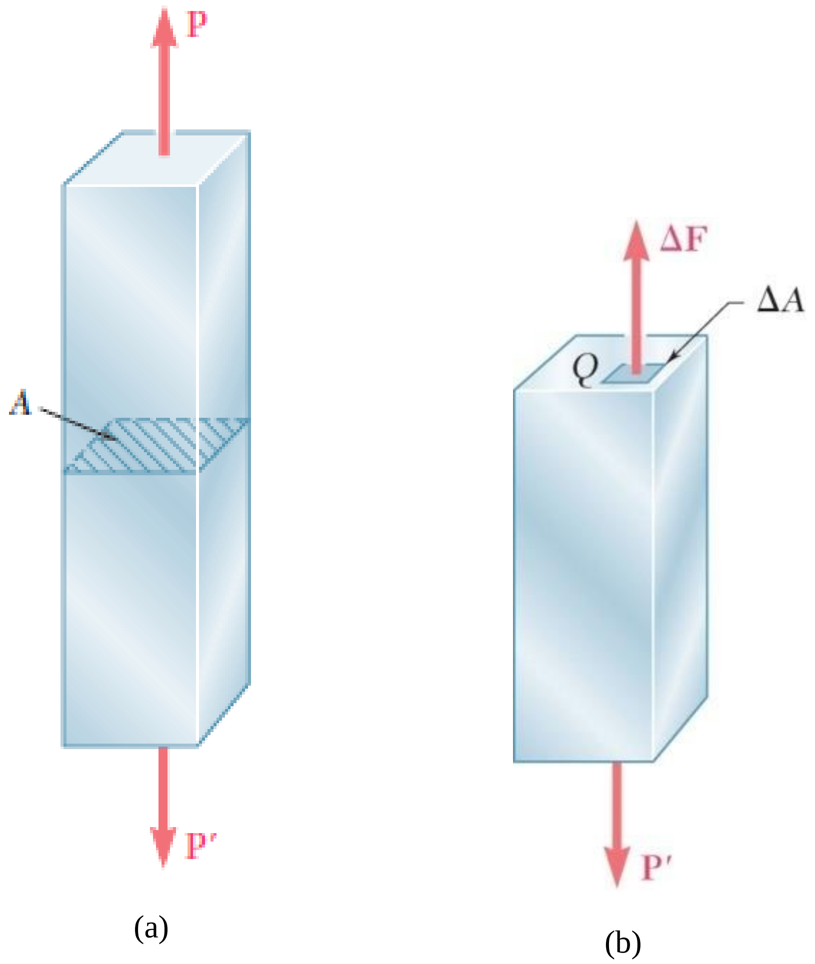


Fig. 1.9 a) axially loaded two-force member b)Small area ΔA, at an arbitrary cross section point carries/axial ΔF in this member.

- *The resultant of the internal forces* for *an axially loaded* member is *normal* to a section cut perpendicular to the member axis.

- The force intensity on that section is defined as the normal stress. σ is obtained by dividing the magnitude P of the resultant of the internal forces distributed over the cross section by the area A of the cross section; it represents, therefore, the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section.

$$\sigma_{ave} = \frac{P}{A}$$

- To define the stress at a given point Q of the cross section, we should consider a small area ΔA. Dividing the magnitude of **ΔF** by ΔA, we obtain the average value of the stress over ΔA. Letting ΔA approach zero, we obtain the stress at point Q:

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

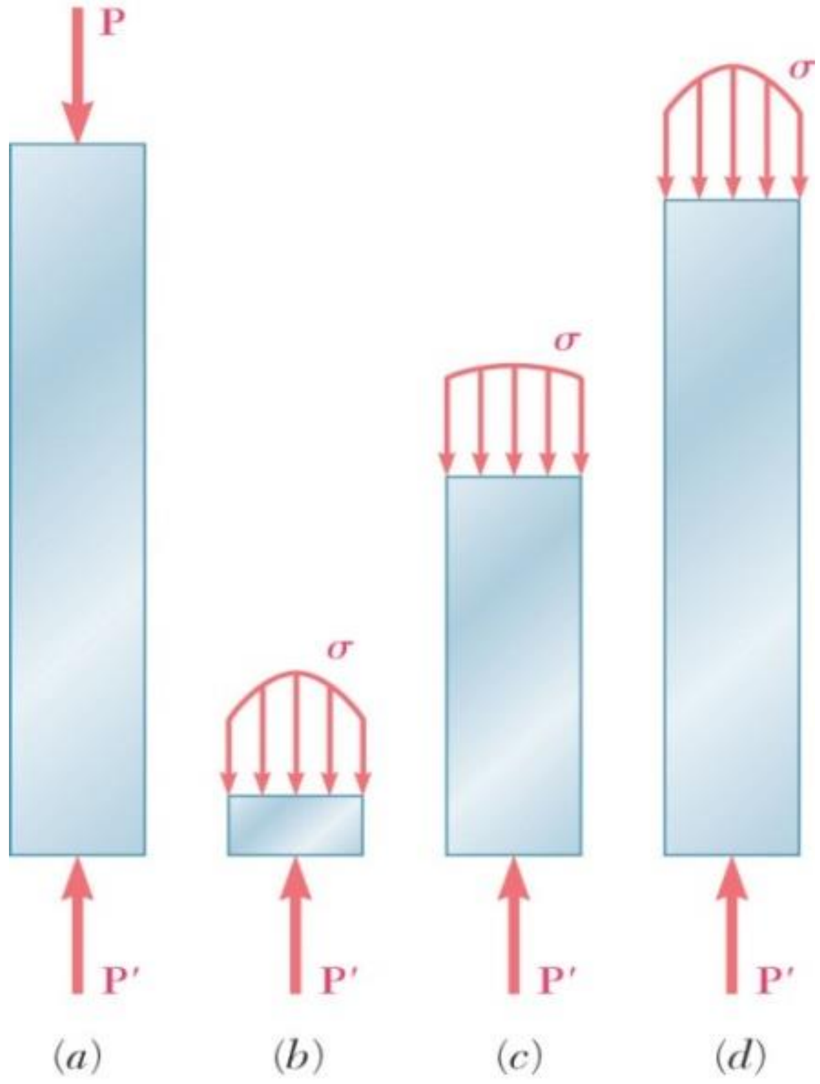


Fig. 1.10 Stress distributions at different sections along axially loaded member.

- The normal stress at a particular point may not be equal to the average stress but the resultant of the stress distribution must satisfy

$$P = \sigma_{ave} A = \int dF = \int_A \sigma \, dA$$

- The actual distribution of stresses is statically indeterminate, i.e., can not be found from statics alone.

- In practice, it will be assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads

Centric & Eccentric Loading

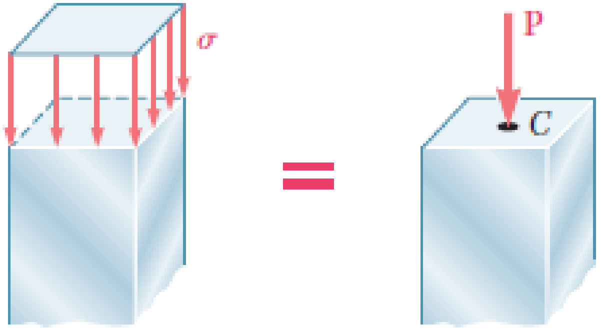


Fig. 1.11.

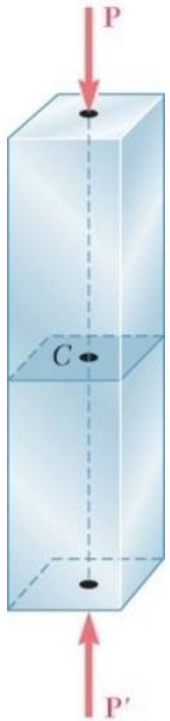


Fig. 1.12 Centric loading having resultant forces passing through the centroid of the section.

- A uniform distribution of stress in a section infers that the line of action for the resultant of the internal forces passes through the centroid of the section.

- A uniform distribution of stress is only possible if the line of action of the concentrated loads **P** and **P′** passes through the centroid of the section considered. This is referred to as *centric loading*.

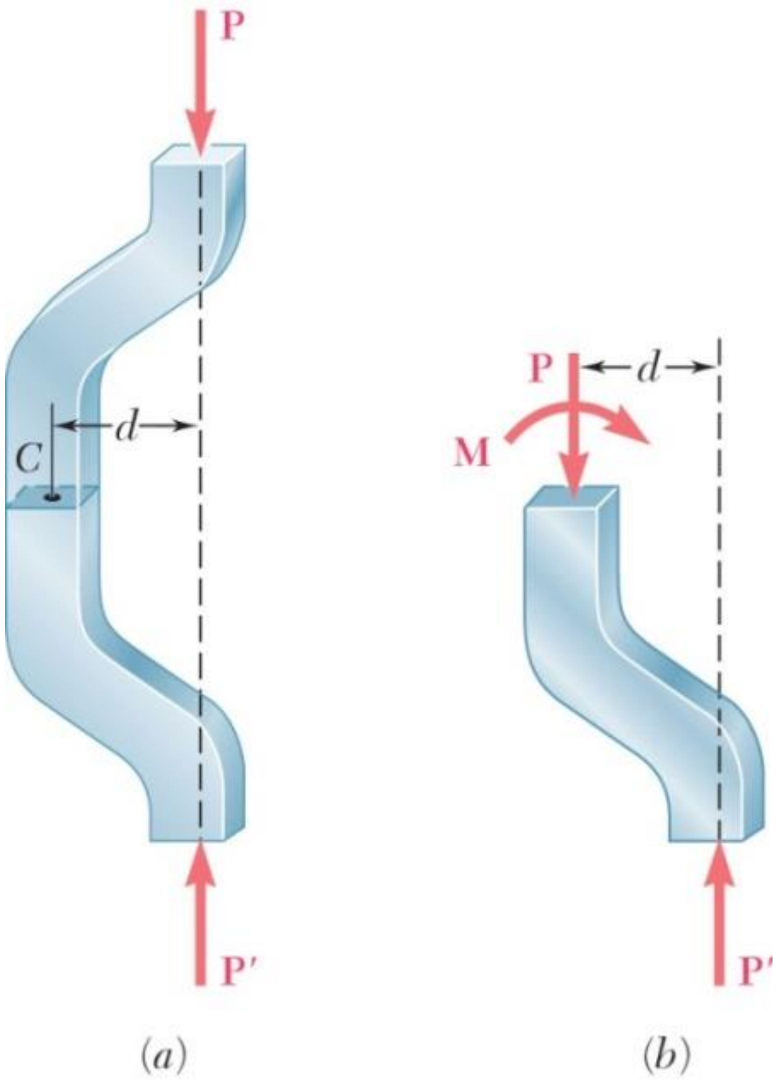


Fig. 1.13 An example of simple eccentric loading.

Centric & Eccentric Loading

- If a two-force member is *eccentrically loaded*, then the resultant of the stress distribution in a section must yield an axial force and a moment.

- The stress distributions in eccentrically loaded members cannot be uniform or symmetric.

Shearing Stress

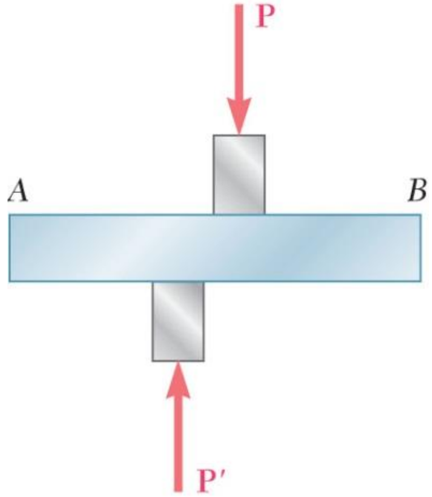


Fig. 1.14 Opposing transverse loads creating shear on member *AB*.

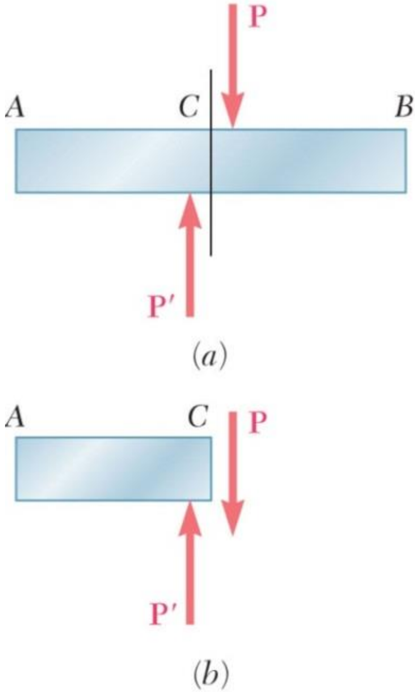


Fig. 1.15 This shows the resulting internal shear force on a section between transverse forces.

- Forces **P** and **P′** are applied transversely to the member *AB*.
- Corresponding elementary internal forces act in the plane of section *C* and are called *shearing* forces.
- The resultant of the internal shear force distribution is defined as the *shear* of the section and is equal to the load **P**.
- The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.
- The shear stress distribution cannot be assumed to be uniform.

Shearing Stress Examples

- Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components.

Single Shear

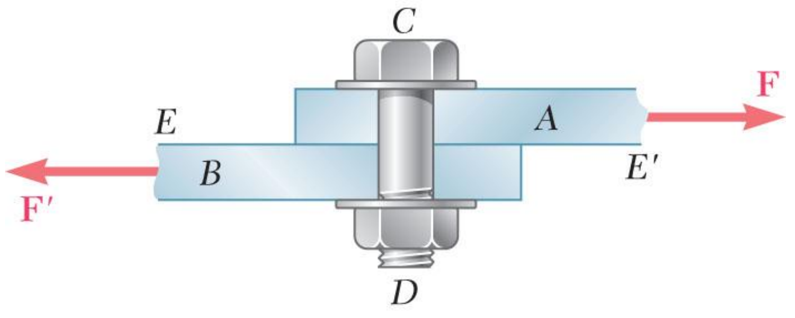


Fig. 1.16 Bolt subject to single shear.

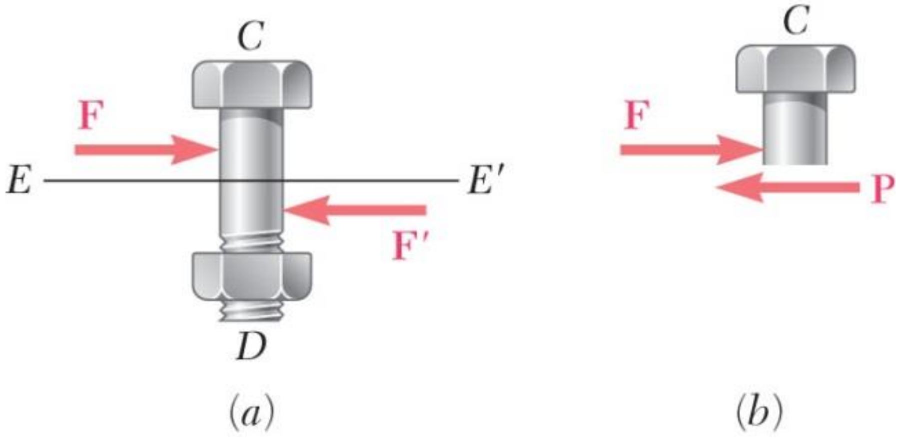


Fig. 1.17 (a) Diagram of bolt in single shear; (b) section *E-E'* of the bolt

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

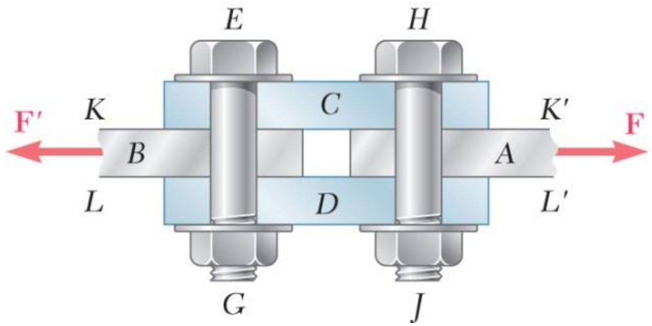


Fig. 1.18 Bolt subject to double shear.

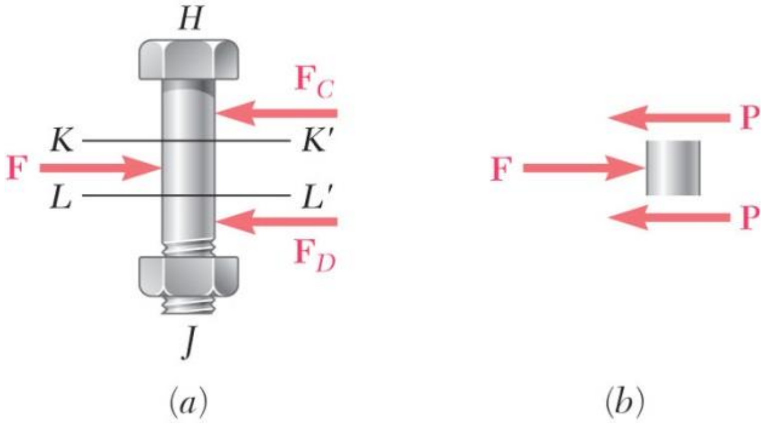


Fig. 1.19 (a) Diagram of bolt in double shear; (b) section *K-K'* and *L-L'* of the bolt.

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

Bearing Stress in Connections

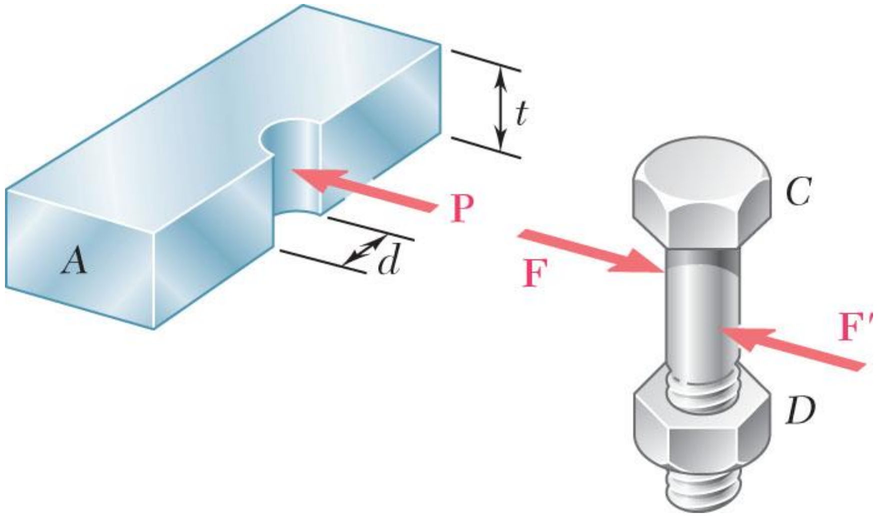


Fig. 1.20 Equal and opposite forces between plate and bolt, exerted over bearing surfaces.

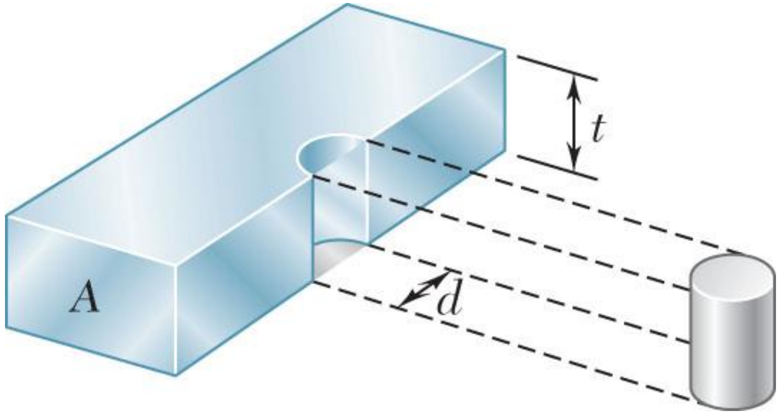


Fig. 1.21 Dimensions for calculating bearing stress area.

- Bolts, rivets, and pins create stresses on the points of contact or *bearing surfaces* of the members they connect.
- The *resultant* of the force distribution on the surface is equal and opposite to the force exerted on the pin.
- Corresponding average force intensity is called the **bearing stress**,

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$